A SIMPLE UNIVERSAL EQUATION FOR GRAIN SETTLING VELOCITY

R.I. FERGUSON[1]* AND M. CHURCH[2]
[1] *Department of Geography, Sheffield University, Sheffield S10 2TN, U.K.*
[2] *Department of Geography, University of British Columbia, Canada*
email r.i.ferguson@durham.ac.uk

ABSTRACT: A new equation is presented for sediment fall velocity as a function of grain diameter for given values of fluid viscosity and fluid and solid density. Sediment fall velocity is a fundamental parameter in the modeling and interpretation of fluviatile and coastal deposition. The equation applies to the entire range of viscous to turbulent conditions, and its simple explicit form makes it easy to use in computer models and other applications in sedimentology, geomorphology, and engineering. The equation is derived from dimensional analysis and converges on Stokes' law for small grains and a constant drag coefficient for large grains. Its two physically interpretable parameters are easily adjusted for shape effects or for the use of sieve diameter rather than nominal grain diameter. It gives a close fit to published and new experimental data for both quartz sand and low-density materials, with no more error than previous equations of more complicated form.

BACKGROUND

The velocity with which particles of specified size settle in water is a fundamental variable in physical sedimentology. The dependence of fall velocity on particle size leads to vertical size sorting when grains settle in standing water, and longitudinal sorting when grains settle from a decelerating current as in deltaic environments. Sorting according to fall velocity also occurs during fluvial transport: depending on the shear velocity of the flow, particles below some critical size travel in suspension whereas larger ones travel as bedload. A quantitative knowledge of how fall velocity varies with sediment size is essential for modeling any of these or similar sorting processes, and for interpreting depositional environments in the rock record.

This knowledge exists for small particles whose settling is dominated by viscous drag, and for large particles whose settling is dominated by inertial forces, but for particles of intermediate size the precise nature of the relation is not agreed. For quartz-density particles settling in water the transitional range spans all but the finest sand, and extends a little way into the gravel range. This is a particularly important range in some applications. For example, the settling of sand from suspension in rivers may be a factor in the often abrupt transition from a gravel bed to a sand bed (Sambrook Smith and Ferguson 1995; Dade and Friend 1998). In this paper we present a simple, explicit equation for fall velocity over the entire size range, including the transitional region, and show that it agrees well with published and new experimental data. The equation is intended to be applicable when all that is known is the sieve size of the grains concerned. Many previous formulations require measurements of individual axis lengths in order to compute the nominal diameter of an equivalent sphere, and in one case also a shape factor. This information is seldom available in large-scale fluvial or oceanographic research and modeling, so a parsimonious approach is highly attractive.

THEORY

The slow settling of small particles is resisted by the viscous drag of the laminar flow around each grain. For solitary spherical particles it follows Stokes' law:

$$w = \frac{RgD^2}{C_1 \nu} \qquad (1)$$

where w denotes the particle's fall velocity, D its diameter, R its submerged specific gravity (1.65 for quartz in water), g the acceleration due to gravity, ν the kinematic viscosity of the fluid (1.0×10^{-6} kg m^{-1} s^{-1} for water at 20°C), and C_1 a constant with a theoretical value of 18. Fall velocity therefore increases as the square of particle diameter. Fall velocities of nonspherical particles, rough particles, or particles in very high concentrations are somewhat lower (i.e., C_1 is larger), as reviewed in standard texts such as Raudkivi (1990). Stokes' law holds for particle Reynolds numbers ($\mathbf{Re} = wD/\nu$) below a value of about 1.

The rapid settling of large particles is resisted predominantly by the turbulent drag of the wake behind each grain. It can be represented by

$$w = \sqrt{\frac{4RgD}{3C_2}} \qquad (2)$$

where C_2 is the constant asymptotic value of the drag coefficient $C_D = 4RgD/3w^2$ at $10^3 < \mathbf{Re} < 10^5$. Numerous experimental investigations have shown that $C_2 \approx 0.4$ for smooth spheres, and $C_2 \approx 1$ for natural grains with some departure from sphericity and possible angularity of edges (see review in Cheng 1997). Equation 2 implies that fall velocity varies as only the square root of particle diameter, instead of the square as in Stokes' law.

Many alternative graphs and empirical equations have been presented for natural grains in the transitional range $1 < \mathbf{Re} < 10^3$, which corresponds to about $0.1 < D < 4$ mm for quartz sand settling in water. The earliest equation we know is by Hallermeier (1981), who proposed $\mathbf{Re} = A^{0.7}/6$ for the range $39 < A < 10^4$, where A is what Yalin (1972) termed the Archimedes or buoyancy index, $A = RgD^3/\nu^2$. This implies $w \propto D^{1.25}$ in the transitional range. Hallermeier used sieve diameter in calculating A and **Re**. In combination with Stokes' law for lower values of A, and $C_D = 1.2$ for higher values, his equation gave an overall root-mean-square (rms) prediction error of ± 15% for a set of 115 experimental values of settling velocity from 13 sources. Van Rijn (1989) suggested that a practical calculation for river sands was to use Stokes' law for $D < 0.1$ mm, a constant C_D of 1.1 for $D > 1$ mm, and $\mathbf{Re} = 10\,[(1 + 0.01A)^{0.5} - 1]$ for the transitional range. Van Rijn's relation is discontinuous in level at the joins of its three parts, and Hallermeier's is discontinuous in slope. Ahrens (2000) tried to avoid discontinuities by suggesting a universal equation of the form $\mathbf{Re} = k_1A + k_2A^{0.5}$ but found that the constants k_1 and k_2 had to be expressed as complicated functions of A to match the fit of Hallermeier's three-part relation.

Dietrich (1982) presented a much more detailed analysis based on > 1000 experimental values from a variety of published sources. He related $w^* = w^3/Rg\nu$ to A (which he called D^*), using nominal diameter to compute A. In a three-stage procedure he fitted $\log w^* = R_1 + R_2 + \log R_3$, where R_1 is the trend for smooth spherical particles, R_2 is a correction for shape effects, and R_3 is a correction for roundness effects. R_1 is a fourth-order polynomial in log A, fitted to the relevant subset of the data plus a large set of low-D points complying with Stokes' law. R_2 is a complicated function of log A and the Corey shape factor (CSF) and was fitted to residuals from R_1. R_3 is a complicated function of log A, CSF, and the Powers roundness index (PRI) and was fitted to residuals from R_2. The full

* Present address: Department of Geography, University of Durham, Durham DH1 3LE, U.K.

JOURNAL OF SEDIMENTARY RESEARCH, VOL. 74, NO. 6, NOVEMBER, 2004, P. 933–937
 1527-1404/04/074-933/$03.00

expression contains 15 empirical coefficients. It gave an excellent fit to the data on which it was calibrated, and is the only relation known to us that makes specific and separate allowance for shape and roundness effects. Dietrich suggested that typical natural grains have CSF = 0.7 (on a 0–1 scale) and PRI = 3.5 (0–6 scale). For quartz in water this gives a reduction in fall velocity (compared to a spherical grain) of 10, 20, or 40% for D = 0.02, 1, or 10 mm according to our calculations. By comparison with Equations 1 and 2 this implies that the ''natural'' values of C_1 and C_2 are 20 and 1.1 respectively, instead of 18 and 0.4 for smooth spheres.

Other researchers, including most recently Cheng (1997), have suggested universal relations between C_D and **Re**. These usually give only implicit relations between w and D, but Cheng's proposal of $C_D = [(32/\mathbf{Re})^{0.67} + 1]^{1.5}$ gives an explicit equation. It applies to natural grains using nominal diameter, and is asymptotic to $C_D = 1.0$ for coarse grains and to Stokes' law with a constant of 24 (not 18) for fine grains. Cheng showed that his relation fitted a selection of experimental data better than several previous universal or multi-part relations in the engineering literature, including that of van Rijn, but his comparison did not include relations proposed by sedimentologists such as Hallermeier (1981) or Dietrich (1982).

Here we present a dimensionally correct explicit equation which reduces asympotically to Equation 1 for small grains and Equation 2 for large ones, can be used for either sieve diameter or nominal diameter, fits published and new experimental data in the transitional range as well as previous relations, but is simpler than any of them.

DIMENSIONAL ANANYSIS AND PROPOSED EQUATION

The relation between fall velocity w [dimensions L T^{-1}] and particle diameter D [L] is expected to depend on the kinematic viscosity ν [L^2T^{-1}] and density $\boldsymbol{\rho}$ [M L^{-3}] of the fluid, and the immersed unit weight $\boldsymbol{\gamma} = (\boldsymbol{\rho}_s - \boldsymbol{\rho})g$ [M L^2T^{-2}] of the sediment. The relation should therefore be fully describable using two nondimensional groups. The standard dimensional analysis of settling takes w and D as nonrepeating variables and leads to the groups w* and D* (or A) used by Dietrich (1982). The alternative pursued here is to take D as a repeating variable, with w and ν nonrepeating. This gives groups $w' = w/(RgD)^{0.5}$ and $\nu' = \nu/(RgD^3)^{0.5}$. These are inversely related to C_D and A, respectively: $w' = (4/3C_D)^{0.5}$ and $v' = 1/A^{0.5}$.

The general relation between w' and ν' must reduce to Equation 1 at small D and Equation 2 at large D: that is, to $1/w' = C_1\nu'$ and $1/w' = (0.75C_2)^{0.5}$ respectively. The simplest possible function which gives these asymptotic limits is their sum:

$$\frac{1}{w'} = C_1\nu' + (0.75C_2)^{0.5} \tag{3}$$

This yields a simple explicit equation for fall velocity as a function of diameter:

$$w = \frac{RgD^2}{C_1\nu + (0.75C_2RgD^3)^{0.5}} \tag{4}$$

in which the parameters C_1 and C_2 take values of 18 and 0.4 for smooth spheres, but somewhat higher values for natural grains as discussed later. The viscosity term in Equations 3 and 4 dominates for small D but becomes negligible for large D. The relation therefore reduces to Stokes' law with constant C_1 for fine sediment but to a constant drag coefficient C_2 for coarse sediment. It can also be expressed as an explicit equation for the drag coefficient:

$$C_D = \left(\frac{2C_1\nu}{\sqrt{3RgD^3}} + \sqrt{C_2}\right)^2 \tag{5}$$

Again the viscosity term dominates for small D but becomes negligible for large D.

Equation 4 is the key result of this paper. It is the simplest possible dimensionally consistent equation for fall velocity in the transitional range and, as we show below, it also gives a close fit to experimental fall velocities.

EVALUATION AND TESTING

We assess the utility of Equation 4 in three stages. The first is to compare it visually with the plots of some previous equations for standard conditions of quartz grains falling in water at 20°C. We then quantify the goodness of fit of Equation 4 and some of its predecessors to experimental data assembled by Raudkivi (1990) and Hallermeier (1981). Thirdly, we report new experiments on the fall velocity of natural river sands and use them for a further test of our proposed equation.

Comparison with Previous Curves and Experimental Data

Equation 4 is plotted in Figure 1A for the case of quartz-density sediment (R = 1.65) settling in water at 20°C and with three combinations of values for the parameters C_1 and C_2: 18 and 0.4 as the limit for smooth spheres, 24 and 1.2 as the likely opposite extreme for angular natural grains, and 18 and 1.0 as a possible intermediate relation for grains of varied shape. The relations proposed by Cheng (1997) and Dietrich (1982) are plotted in Figure 1B for comparison, with separate curves for the spherical and natural versions of Dietrich's relation. To facilitate comparison, both plots include the expected asymptotic trends for spherical particles (Stokes' law and a constant C_D of 0.4) and some published experimental data for ''naturally worn'' quartz grains with CSF = 0.7 (Raudkivi 1990, Table 2.2, based on a U.S. Federal Inter-Agency Committee report). The data are reported using nominal grain diameter, as assumed in most of the published predictive equations.

Figure 1A shows that for diameters below 0.1 mm and above 2 mm our equation is indistinguishable from power laws with slopes of 2 and 0.5, respectively, in accordance with Stokes' law and a constant drag coefficient. An increase in C_1 pulls the lower left end of the curve down, and an increase in C_2 pulls the top right end down. With $C_1 = 18$ and $C_2 = 0.4$ it is asymptotically the same as Dietrich's relation for spheres (Fig. 1B) at the fine and coarse ends of the size range, but it plots slightly higher in the transitional region. With $C_1 = 24$ and $C_2 = 1.2$ it plots slightly below Dietrich's ''natural'' relation for all sizes, though more so at the fine end; conversely, it is asymptotically the same as Cheng's relation for small grains but plots slightly lower for medium and large grains, because Cheng's asymptote is equivalent to $C_2 = 1.0$. With the intermediate values $C_1 = 18$ and $C_2 = 1.0$ our equation plots slightly higher than Cheng's for all sizes below about 8 mm, and slightly higher than Dietrich's ''natural'' relation in the range 0.08–0.8 mm, but very close to Dietrich's for sizes below 0.08 mm and above 0.8 mm. We noted above that Dietrich's ''natural'' relation for CSF = 0.7 is asymptotically equivalent to $C_1 = 20$ and $C_2 = 1.1$ in our terms. With these parameter values our relation is visually almost indistinguishable from his, with predictions that are never more than 4% lower or 8% higher.

The experimental data lie between the extreme curves in each plot, as would be expected. The fit of our and previous equations to the data in Figure 1 is summarized in the first three columns of Table 1, and it throws some light on the question of assigning values to C_1 and C_2. We assess fit here and later by the root-mean-square prediction error (rmse), defined as the standard deviation of the percentage prediction error for each data point, $100(p - o)/o$, where p and o denote predicted and observed fall velocity. Hallermeier's and Ahren's equations are for sieve diameter, not nominal diameter; the nominal grain diameters were therefore reduced by 10%,

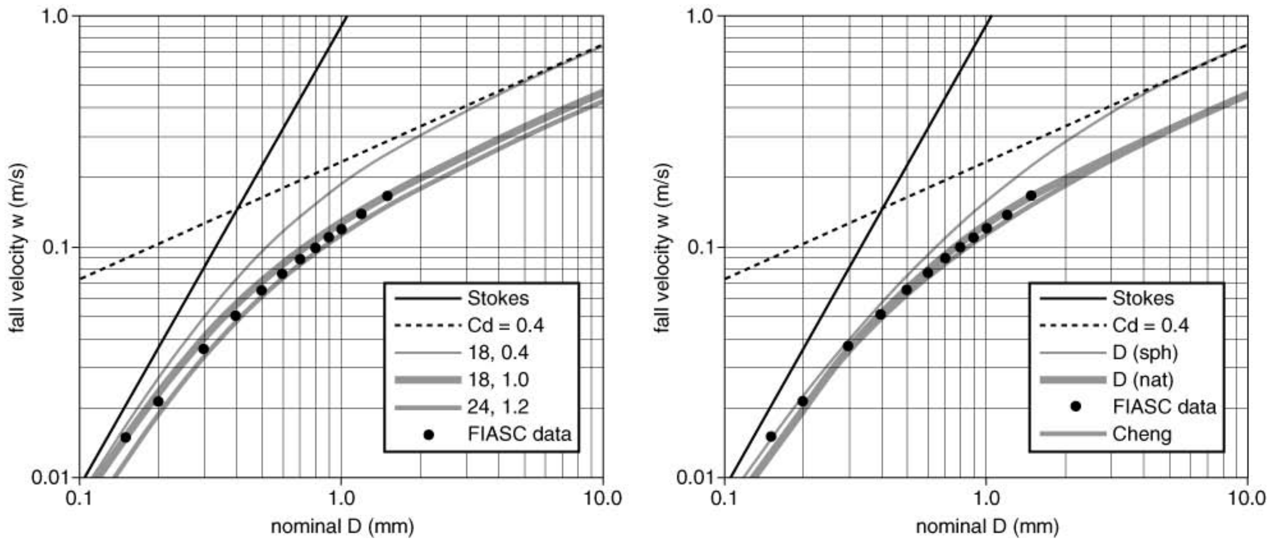

FIG. 1.—Predicted relation between fall velocity and diameter for quartz grains in water at 20°C according to **A)** Equation 4 and **B)** previous authors. Straight lines in both plots show expected asymptotic trends for smooth spheres (Stokes' law with $C_1 = 18$, and constant drag coefficient $C_2 = 0.4$). Points labeled FIASC are experimental values from the U.S. Federal Inter-Agency Sedimentation Conference as listed in Raudkivi (1990). Upper, middle, and lower curves in Part A are for spherical, natural, and angular grains using C_1, C_2 shown in legend. Upper and middle curves in Part B are Dietrich's (1982) relation for spheres and natural grains, respectively; lower curve is Cheng's (1997) equation.

which is the typical difference between the different definitions for grains of CSF $\approx$ 0.7 according to Raudkivi (1990, p. 18).

As would be expected, when parameter values for smooth spheres are used in our and Dietrich's equations the fit to data for natural grains is poor, with considerable overprediction at all grain sizes. At the other extreme, Cheng's equation and our equation with ''angular'' values underpredict systematically for small and large grains, but their good fit for intermediate sizes keeps the rmse below 10% in each case. Our equation with ''intermediate'' parameter values overpredicts the experimental fall velocities in the middle of the data range but again does moderately well overall with rmse below 10%. The best fits are by the relations of Ahrens (rmse only 2%), Hallermeier (5%), and Dietrich (5% with ''natural'' parameter values) and by our equation with Dietrich-equivalent values of $C_1 = 20$ and $C_2 = 1.1$ (rmse 4%). This confirms that our equation gives results comparable with the best previous equations when the parameter values used are compatible with the shape characteristics of the natural grains in the experiments.

For a second test of predictive ability we use the data assembled by Hallermeier (1981) from several previous sources. These data relate to the settling of quartz sand and lower-density natural materials, such as pumice, in water at various temperatures. The data set is much bigger than Raudkivi's ($n = 115$), and the range of densities and temperatures provides a check on the way these variables are incorporated in the new dimensional analysis underlying our relation. Grains in this data set were stated by Hallermeier to be ''somewhat angular'' but no quantitative measures of shape or roundness are reported. Grain diameters in this data set are based on sieving, so were increased by 10% to estimate nominal diameters for equations using these. A graphical comparison like Figure 1 is no longer possible because of the variety of densities and viscosities in this data set, but we report goodness of fit in the right-hand columns of Table 1.

TABLE 1.—*Mean and root-mean-square (rms) percentage errors in predicting experimental values of fall velocity.*

Prediction Equation	Data from Raudkivi (1990)		Data from Hallermeier (1981)	
	Mean Error	Rms Error	Mean Error	Rms Error
Hallermeier (1981)	1	5	1*	15*
Dietrich (1982): spheres	19	23	25	40
Dietrich (1982): natural	−1	5	1	17
Van Rijn (1989)	9	12	8	22
Cheng (1997)	−7	7	−4	16
Ahrens (2000)	−1	2	1*	15*
Eq. 4: $C_1 = 18$, $C_2 = 0.4$	45	47	33	44
Eq. 4: $C_1 = 18$, $C_2 = 1.0$	7	8	−1	16
Eq. 4: $C_1 = 20$, $C_2 = 1.1$	0	4	−7	17
Eq. 4: $C_1 = 24$, $C_2 = 1.2$	−8	9	−15	22

A positive mean error represents overprediction. Errors are starred (*) if the experimental data were sued to calibrate parameters in the prediction equation.

The relations of Hallermeier (1981) and Ahrens (2000) were calibrated to these data and not surprisingly give the best fits (rmse 15% in each case), but several other equations perform almost as well. Those of Cheng (1997) and Dietrich (1982, ''natural'' parameters) give rmse 16% and 17%, respectively. Our equation 4 using sieve diameters and the ''intermediate'' parameter values $C_1 = 18$ and $C_2 = 1.0$ also gives a good fit to these data, with rmse 16%. The predictions lie close to the 1:1 line when plotted against measurements (Fig. 2), and the correlation between logarithms of observed and predicted values exceeds 0.99 despite the presence of one outlier which all equations underpredict by about 60%. The good fit of Equation 4 to Hallermeier's data, which contain a wide range in particle density and a moderate range in viscosity, suggests that our novel dimensional analysis correctly captures the effect of these grain and fluid conditions. If nominal diameter is used in our equation, the fit for parameter values 18 and 1.0 is slightly less good (rmse 17%) but that for Dietrich-equivalent values (20 and 1.1) improves (rmse 16%). In summary, for Hallermeier's (1981) data as well as Raudkivi's (1990) data, the new equation has predictive ability similar to that of the best previous relations.

Comparison with New Experimental Data

As a further test of our proposed relation we have performed new settling-column experiments with natural sand grains falling in water at 23–24°C. The purpose of this work was to obtain data with high precision in both fall velocity and grain diameter, by making replicate measurements on grains from narrow sieve classes. We chose to define grain diameter in terms of sieve meshes because our goal is a simple equation that can be used in larger-scale studies of rivers, including computer models. Such

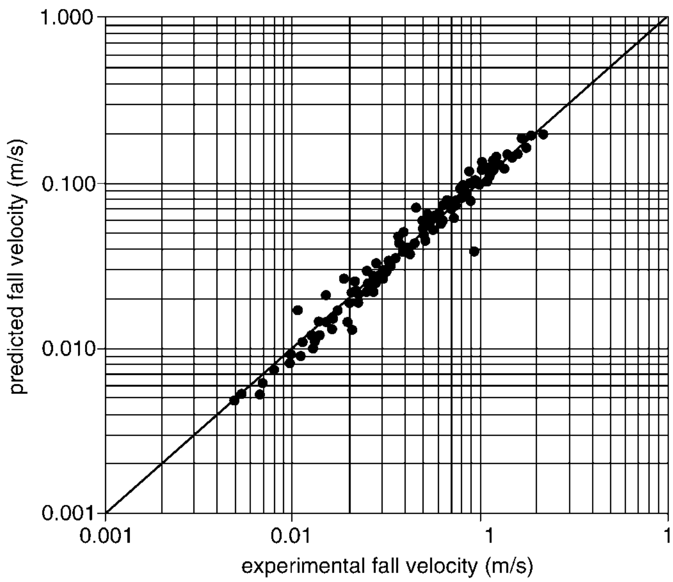

FIG. 2.—Agreement between experimental data of Hallermeier (1981) and predictions of fall velocity using Equation 4 with ''natural'' coefficient values ($C_1 = 18$, $C_2 = 1.0$).

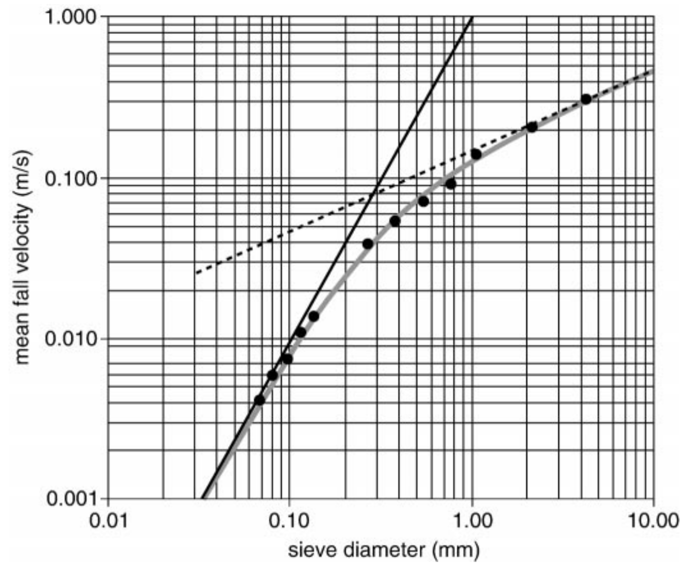

FIG. 3.—Predicted relation between fall velocity and sieve diameter, using $C_1 = 18$ and $C_2 = 1.0$ in Equation 4, compared to new experimental data for natural river sands (see text for details). Straight lines are the two asymptotes of the equation: Stokes' law and a constant drag coefficient of 1.0.

studies normally characterize sediment mixtures in terms of bulk composition by sieve size, not on the basis of individual grain-axis measurements. The grains were obtained from bulk samples of bed material in the lower Fraser River in western Canada. They were mainly quartz or feldspar but with a few darker minerals. We made a large number of replicate measurements of w for each of 12 narrow (quarter-phi) sieve size classes ranging from 0.062–0.074 mm to 4.0–4.8 mm. For the three coarsest sizes, individual grains of compact shape were timed by pairs of students over a settling distance of 1.0 m in a cylinder of length 1.2 m. The internal diameter was 0.14 m, which is an order of magnitude larger than necessary to avoid wall drag effects. The experimenters did not know the theoretically expected outcome. A total of 47 measurements were made for each size class. The experiments on sizes below 1 mm were made in the same cylinder by the second author, with 60 replicates per size class. A few tens of grains were introduced, and one grain near the center of the cloud (and thus presumably neither of high density nor strongly aspherical) was selected for timing over a distance of 0.2 to 0.6 m, depending on how low the start line for timing had to be for grains to reach terminal velocity.

The mean settling time for each size was calculated, and also the standard deviation. Grains falling outside ± 2 standard deviations were then excluded, and the statistics recalculated, in an iterative procedure until all grains were within this tolerance. This excluded only a few grains except in the two finest classes. The excluded grains either fell more slowly than the others with visible oscillations suggestive of a flattened shape, or faster than the others presumably because they were of higher density than quartz. The high degree of replication, avoidance of the leading edge of the group of grains, and subsequent elimination of outliers were collectively meant to avoid spuriously high fall velocities associated with heavy minerals, but we cannot be absolutely certain that we succeeded completely in this. The mean fall velocities after this quality-control procedure, and the standard errors of these means, are listed in Table 2. The iterative deletion of outliers had very little effect on the mean values but reduced the standard errors to only 1 or 2% of the mean. By assuming a uniform distribution of grain size in phi units within each sieve class, the standard error of each D value is less than 1%.

Figure 3 shows the fit of Equation 4 with its ''intermediate'' coefficients to these new measurements. The curve is plotted for a submerged specific gravity of $R = 1.65$ and a kinematic viscosity of $\boldsymbol{\nu} = 9.2 \times 10^{-7}$ m^{2}s^{-1}. It fits the data extremely well. Seven data points are slightly above the curve, four slightly below. Error bars are not plotted because they are too small for clarity, but we can report that the 95% confidence boxes for seven points intersect the curve, three are just above it, one just below, and one substantially below ($D = 0.77$ mm, with a prediction error of +13%). The rms error is ± 6%.

Table 3 compares the goodness of fit of alternative equations to the new

TABLE 2.—*New experimental measurements of fall velocity of natural river sands.*

D (mm)	# of Values	w (m s^{-1}) ± Standard Error
0.068	46	0.0425 ± 0.00009
0.081	42	0.0060 ± 0.0001
0.096	55	0.0075 ± 0.0001
0.115	54	0.0110 ± 0.0002
0.136	54	0.0139 ± 0.0001
0.273	53	0.0388 ± 0.0002
0.386	57	0.0551 ± 0.0005
0.55	52	0.0729 ± 0.0010
0.77	58	0.0930 ± 0.0016
1.09	42	0.141 ± 0.002
2.18	44	0.209 ± 0.002
4.36	42	0.307 ± 0.003

See text for details of experiments. D denotes geometric-mean sieve diameter.

TABLE 3.—*Mean and root-mean-square (rms) percentage errors in predicting new experimental values of fall velocity.*

Equation	Mean Error	Rms Error
Hallermeier (1981)	7	13
Dietrich (1982): spheres	26	33
Dietrich (1982): natural	2	7
Van Rijn (1989)	18	22
Cheng (1997)	−4	6
Ahrens (2000)	4	9
Eq 4: $C_1 = 18$, $C_2 = 0.4$	27	35
Eq 4: $C_1 = 18$, $C_2 = 1.0$	−1	6
Eq 4: $C_1 = 20$, $C_2 = 1.1$	−8	10
Eq 4: $C_1 = 24$, $C_2 = 1.2$	−17	19

A positive mean error represents overprediction.

data in the same way as in Table 1. The equation of Ahrens (2000), which fitted Raudkivi's data best of all (Table 1), also performs well on the new data but gives a rather higher rmse (9%) than Equation 4. The equations of Dietrich (1982; ''natural'' parameters) and Cheng (1997) also fit the data well (rmse 6% and 7%, respectively) once sieve diameters are increased by 10% as estimates of nominal diameter. But once again these excellent fits by previously published equations are matched by our equation either as above or using nominal diameter and Dietrich-equivalent parameters ($C_1 = 20$, $C_2 = 1.1$; rmse 6%).

CONCLUSIONS

Several authors have suggested universal or multi-part relations between fall velocity *w* and particle diameter *D* that span the transitional size range in which both viscous and inertial forces are important. For quartz-density particles in water this range is from fine sand to granules. We have derived a simple explicit formula (Equation 4) for all grain sizes, including the transitional range, from a new dimensional analysis of the problem together with the assumptions that the relation must reduce to Stokes' law for fine sediment and a constant drag coefficient for coarse sediment. The proposed equation is dimensionally correct and includes the effects of viscosity and submerged specific gravity. It contains only two coefficients, which is fewer than any previous relation, and both of them are physical parameters rather than empirical ''fudge factors'' as in most other equations. One (C_1 in Equation 4) is the constant in Stokes' equation for laminar settling; the other (C_2) is the constant drag coefficient for particle Reynolds numbers exceeding 10^3. For smooth spheres these parameters take values 18 and 0.4, respectively. For typical natural sands we suggest 18 and 1.0 where sieve diameters are used, or 20 and 1.1 where nominal diameters are used. These values give excellent fits to two existing experimental data sets and one new set, all for natural sands of nonspherical shape. A likely limit for very angular grains is $C_1 = 24$, $C_2 = 1.2$.

An important advantage of the new equation for many purposes is its simplicity. This makes it easy to incorporate in computer models of larger-scale sediment-transport and sedimentation problems. A second advantage is that the equation can be used with either sieve diameters or nominal diameters, by adjusting the parameter values slightly as suggested above. The equation may be useful in a range of applications in physical sedimentology and engineering.

ACKNOWLEDGMENTS

Students in undergraduate classes over several years at the University of British Columbia performed the experiments on the coarsest sizes. Reviews by Bill Dietrich and Peter Wilcock led us to explain more fully the previous literature and the UBC experiments, and to discuss the choice of parameter values in more detail, but confirmed our belief in the value of a simple relation with physically interpretable coefficients.

REFERENCES

Ahrens, J.P., 2000, A fall-velocity equation: Journal of Waterway, Port, Coastal, and Ocean Engineering, American Society of Civil Engineers, v. 126, p. 99–102.

Cheng, N.-S., 1997, Simplified settling velocity formula for sediment particle: Journal of Hydraulic Engineering, American Society of Civil Engineers, v. 123, p. 149–152.

Dade, W.B., and Friend, P.F., 1998, Grain-size, sediment-transport regime, and channel slope in alluvial rivers: Journal of Geology, v. 106, p. 661–675.

Dietrich, W.E., 1982, Settling velocity of natural particles: Water Resources Research, v. 18, p. 1615–1626.

Hallermeier, R.J., 1981, Terminal settling velocity of commonly occurring sand grains: Sedimentology, v. 28, p. 859–865.

Raudkivi, A.J., 1990, Loose Boundary Hydraulics, 3rd edition: Oxford, U.K., Pergamon, 533 p.

Sambrook Smith, G.H., and Ferguson, R.I., 1995, The gravel–sand transition along river channels: Journal of Sedimentary Research, v. A65, p. 423–430.

Van Rijn, L.C., 1989, Handbook on Sediment Transport by Currents and Waves: Delft, The Netherlands, Delft Hydraulics, Report H461.

Yalin, M.S., 1972, Mechanics of sediment transport: Oxford, U.K., Pergamon, 290 p.

Received 28 January 2004; accepted 12 May 2004.